UNIVERSITÉ DE MONTRÉAL

**CENTRE DE RECHERCHE SUR LES TRANSPORTS**

CASE POSTALE 6128, STATION A, MONTRÉAL, QUÉBEC, H3C 3J7

# DETERMINING MODAL CHOICE AND STATION CHOICE FOR A RAIL TRANSIT LINE

by

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This paper was presented at the Session on Transit Networks of the Joint National ORSA/TIMS Meeting, November 7, 1977, Atlanta, Georgia, U.S.A.

This work was supported in parts by grants from the Research and Development Centre of Transport Canada, and the F.C.A.C. program of the Ministère de l'Education du Québec.

Université de Montréal

Publication # 85 - Centre de recherche sur les transports
Cahier # 7711 - Département de sciences économiques

Octobre 1977 - Révisé en mai 1978

Dépôt légal - 3e trimestre 1978

## ABSTRACT

This paper considers the problem of predicting station choice and modal choice for composite mode trips that involve a trip by private car to a rail transit station and the continuation of the trip to the final destination by a rail transit mode. The choice between car and transit, and between park'n ride and kiss'n ride are both taken into account. One of the models presented takes explicitly into account traffic congestion. The methodology proposed relies on entropy maximization with inequality constraints related to parking capacities at the rail transit stations. The numerical application of one of the models to the Lindenwold High Speed Line is reported.



## RÉSUMÉ

Cet article présente des modèles dont l'objectif est la prévision du choix modal et du choix de la station pour des voyages à mode mixte comprenant un voyage par automobile jusqu'à une station de chemin de fer et la continuation du voyage par chemin de fer jusqu'à la destination finale. Le choix entre l'automobile et le chemin de fer ainsi que le choix du mode d'accès (park'n ride versus kiss'n ride) sont tous deux pris en compte. L'un des modèles présentés tient compte aussi de la congestion du trafic. La méthodologie proposée est la maximisation de l'entropie sous des contraintes d'inégalité liées aux capacités des parcs de stationnement des stations de chemin de fer. L'article présente les résultats du test de l'un des modèles à des données sur la Lindenwold High Speed Line à Philadelphie.

## I Introduction

In the recent decade much attention has been devoted in North America to the possibility of developing public transit as an alternative to the private automobile in urban transportation. The park'n ride concept has been considered as an appealing method to attract commuters away from the automobile and towards rail transit. An early application of this concept is the Lindenwold High Speed Line in the Philadelphia area. This line was actually the object of an intensive scrutiny resulting in a series of studies[1] which can be considered as pioneering efforts on the subject of the impact of a modern rail transit line on commuters' travel choices. This new urban transportation concept raises many questions concerning the relationship between parking policy and rail transit usage. These questions are at the origin of this paper, which can be considered as a sequel to a previous paper by Florian and Los [14], which was concerned with mixed mode travel in general: this includes for instance travel by car to a downtown parking lot and continuation of the trip by foot to the final destination.

This paper presents models than can be used to predict the impact on station choice or modal choice of policy measures such as changes in parking capacity, addition or suppression of stations, changes in parking fees. These models, to be useful, must consider kiss'n ride as well as park'n ride as alternative modes of access to the rail transit line [Fig. 1]. The models developed in this paper are based on entropy maximization methodology and take explicitly into account parking lot capacities.

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1. See [2], [6], and [7] for instance.

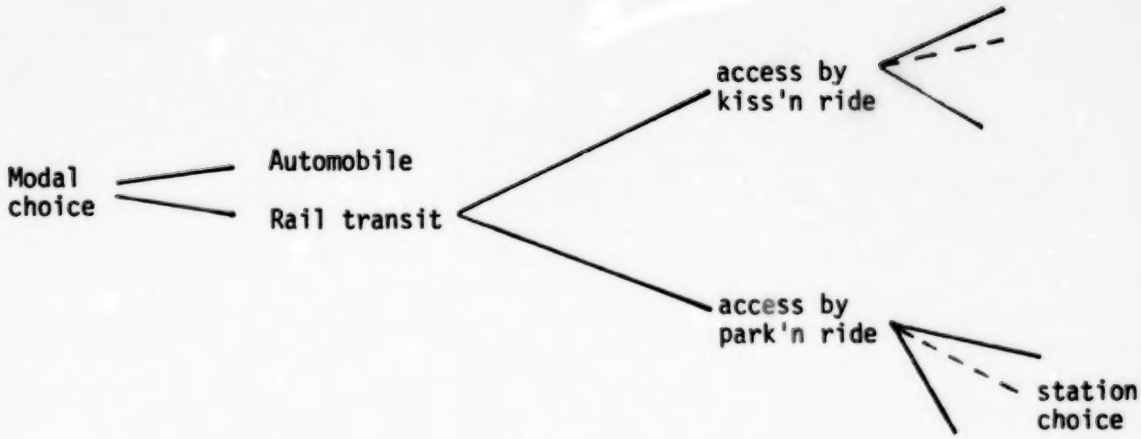


Figure 1: Decisions open to the trip-maker

As mentioned by Florian and Los [14], previous approaches for modelling the attraction of parking lots were unsuccessful. Disaggregate demand models (Cf. [6], [16] and [24]) do not consider capacities explicitly and their use may result in demand predictions which exceed these capacities. Linear programming based parking lot allocation models determine the choice of parking lots as a result of the minimization of an aggregate travel cost. [1, 25]. Although these models take into account parking lot capacities explicitly, the behavior of trip makers is treated in a crude way. Trip distribution models of the gravity type have also been proposed [20] but with equality constraints. Thus parking lot capacities must always be filled and one does not allow for the possibility of underutilizing a parking lot.

To summarize, these previous approaches have failed to model in a satisfactory way the interaction between the demand and supply for parking in a spatial context in general; a fortiori in the particular context of sta-

tion choice and mode choice for a rail transit line. The approach proposed in Florian and Los [14] was to modify the entropy based trip distribution models to consider inequality constraints instead of equality constraints. This approach is followed here and extended to take into account modal choice and traffic congestion on the automobile network.

The paper is organized as follows: In section II, one of the two models proposed in [14] is restated. The solution method is described in Section III. This model predicts station choice (or equivalently parking lot choice) in a park'n ride context. A particular case of this model was implemented and tested on park'n ride data gathered by the Lindenwold High Speed Line Study and the results of the numerical application are presented in section IV. In Section V not only station choice is predicted but modal choice as well: park'n ride versus kiss'n ride for the mode of access to the transit line and car versus transit. Two models are presented for that purpose. The first assumes that there is no congestion on the automobile network, while the second assumes that there is congestion. Section VI concludes the paper.

## II A Model for predicting station choice in a park'n ride situation

The entropy maximization methodology can be justified directly on information theoretical grounds [28]. Alternatively it has been shown by Cochrane [4], Champernowne *et al.* [3] and Williams [26] that formally similar models can be obtained directly from individual random utility models if one makes specific assumptions on the distribution of random utilities in the population studied. It has also been shown by Champernowne *et al.* [3] or

Williams [26] that the problem stated below can be interpreted as maximisation of a "locational surplus".

We assume known the number of person trips originating at i, $O_i$, $i \in I$ and the number of person trips destined to final destination j, $D_j$, $j \in J$. Each trip is taken first by car to a parking lot j, $k \in K$, situated at a station of the rail transit line, and by transit to the final destination. The in-vehicle travel cost per person, measured in generalized cost units, from origin i to parking lot k is $c_{ik}$ and $c_{kj}$ is the transit cost between parking lot k and final destination (it can be the sum of the transit fare and of the value of time spent travelling by the transit mode). Let $\Pi_k$ be the cost of parking at parking k (e.g. the parking fee). The average car occupancy factor $\gamma_k$, may vary for each parking lot k. Let $c_{ikj} = c_{ik} + \Pi_k + c_{kj}$ be the total cost per person travelling from i to final destination j. $T_k$ is the capacity of parking lot k. We require $f_{ikj}$, the number of person-trips that originate at i, use station k (and therefore the parking k) and are destined for j to be the solution to

$$(M1): \text{Min} \sum_i \sum_k \sum_j f_{ikj} \{\ell n\, f_{ikj} + \theta\, c_{ikj}\} \tag{1}$$

$$\text{s.t.} \sum_k \sum_j f_{ikj} = O_i \qquad \text{all i} \tag{2}$$

$$\sum_i \sum_k f_{ikj} = D_j \qquad \text{all j} \tag{3}$$

$$\sum_i \sum_j f_{ikj} \leq \gamma_k T_k \qquad \text{all k} \tag{4}$$

$$f_{ikj} \geq 0 \tag{5}$$

The parameter $\theta$ is assumed known for prediction purposes. In the random utility derivation of entropy models (Cf [4] and [26]), it is inversely

related to the standard deviation of the distribution of random utilities in the population. To assume that $\theta$ is constant between two periods, the calibration and prediction periods say, means that one assumes that the underlying causes of the dispersion of individual preferences do not change. Constraints (2) and (3) are the usual origin-destination constraints. Constraint (4) requires that the parking lot capacities are not exceeded. When $\theta$ tends towards infinity it has been shown [4 and 29] that the solution of (M1) tends towards the solution of the linear programming problem (LPM1):

$$\text{Min} \sum_i \sum_k \sum_j f_{ikj}\, c_{ikj} \tag{6}$$

s.t. (2), (3), (4) and (5)

Thus this model can be considered as an extension of gravity models or of linear programming models. We recall that the objective function is convex in $f_{ikj}$ and we set $f_{ikj} \ln f_{ikj} = 0$ at $f_{ikj} = 0$. The Kuhn-Tucker conditions are necessary and sufficient conditions for the optimal $f^*_{ikj}$. After a few suitable modifications the Kuhn-Tucker conditions become

$$f^*_{ikj} = \exp(-v_k) \exp(-\alpha_i - \beta_j - 1) \exp(-\theta\, c_{ikj}) \tag{7}$$

The station choice, for a given origin-destination pair is given by:

$$\frac{f^*_{ikj}}{\sum_k f^*_{ikj}} = \frac{\exp(-v_k) \exp(-\theta\, c_{ikj})}{\sum_k \exp(-v_k) \exp(-\theta\, c_{ikj})} \tag{8}$$

In addition we have the complementarity conditions

$$v_k > 0 \Rightarrow \gamma_k T_k - \sum_i \sum_j f^*_{ikj} = 0 \tag{9}$$

$$\gamma_k T_k - \sum_i \sum_j f^*_{ikj} > 0 \Rightarrow v_k = 0 \tag{10}$$

If no parking lot were used at full capacity, then $v_k = 0$ for all $k \in K$ and

the parking allocation (8) would reduce to a logit model

$$\frac{f^*_{ikj}}{\sum_k f^*_{ikj}} = \frac{\exp(-\theta\, c_{ikj})}{\sum_k \exp(-\theta\, c_{ikj})} \qquad (11)$$

Otherwise the Lagrange multipliers $v_k$ correspond to the shadow prices associated with the parking constraints and the term $\exp(-v_k)$ "weights" the simple logit allocation.

## III Solution method

For a given value of $\theta$, the model (M1) is a minimization problem with a convex objective function and linear constraints which can be solved by adapting a suitable algorithm such as the Frank and Wolfe algorithm [15] and [30]). Thus it is proposed first to calibrate $\theta$ on observed data related to parking lot (or station) usage. It is assumed that a parking lot survey is available which permits to determine the actual utilization of each station in the model. Let $\hat{T}_k$ be the actual observed usage of station k.

For calibration purposes we replace inequality (4) by equality

$$\sum_i \sum_j f_{ikj} = \gamma_k \hat{T}_k \qquad (4')$$

The model thus obtained is a "triply constrained" gravity model. It can be shown [14] that the maximum likelihood estimate of $\theta$ in that case is the value chosen so that the mean trip cost given by the model

$$C(\theta) = \frac{1}{F} \sum_i \sum_k \sum_j c_{ikj} \cdot f_{ikj}(\theta) \quad \left(F = \sum_i O_i = \sum_j D_j\right)$$

equals the observed mean trip cost $\hat{C} = \frac{1}{F} \sum_i \sum_k \sum_j c_{ikj}\, \hat{f}_{ikj}$

It can also be shown [14] that $C(\theta)$ is a strictly decreasing function of $\theta$. These two results generalize the results obtained by A. Evans for the doubly

constrained gravity model [8]. The monotonicity property of $C(\theta)$ is the basis of many calibration algorithms [27, 19]. In particular, one can use a dichotomic search procedure.

For prediction purposes we assume that $\theta$ has been calibrated. We adapt the Frank and Wolfe method described in appendix 1 to the solution of problem (M1). An iteration of the algorithm requires the solution of a linear program that determines the descent direction and the solution of a one-dimensional minimization problem of a convex function that determines the step size that achieves the best improvement of the objective function. In an intermediate stage of the algorithm a feasible solution $\bar{f}_{ikj}$ is known, The descent direction is obtained by first solving the subproblem:

$$\text{Min} \sum_i \sum_k \sum_j y_{ikj} [1+\ell n\, \bar{f}_{ikj} + \theta\, c_{ikj}] \tag{12}$$

s.t.
$$\sum_k \sum_j y_{ikj} = O_i \qquad \text{all } i \tag{13}$$

$$\sum_i \sum_k y_{ikj} = D_j \qquad \text{all } j \tag{14}$$

$$\sum_i \sum_j y_{ikj} \leq \gamma_k T_k \qquad \text{all } k \tag{15}$$

$$y_{ikj} \geq 0 \tag{16}$$

This problem is a linear program that can be solved by any of the packages available. In the particular case when there is only one final destination j (say the CBD), this linear program reduces to a transportation problem for which special purpose codes exist [17 and 18]. Let $[\bar{y}_{ikj}]$ be the optimal solution of (12) - (16). The optimal step length $\lambda$ is obtained by setting

$$x_{ikj} = \bar{f}_{ikj} + \lambda[\bar{y}_{ikj} - \bar{f}_{ikj}] \tag{17}$$

and finding $\lambda$ that minimizes

$$\sum_i \sum_k \sum_j x_{ikj}[\ell n\ x_{ikj} + \theta\ c_{ikj}] \tag{18}$$

Hence the steps for the algorithm

Step 1: Select an initial solution $(\bar{f}_{ikj})$.

Step 2: Determine the descent direction by solving the linear program (12) to (16) and by setting the direction equal to $(\bar{y}_{ikj} - \bar{f}_{ikj})$.

Step 3:

$$\text{If } \left|\sum_i \sum_k \sum_j [1+\ell n\ \bar{f}_{ikj} + \theta\ c_{ikj}]\ [\bar{y}_{ikj} - \bar{f}_{ikj}]\right| \leq \varepsilon \tag{19}$$

for a suitable convergence parameter $\varepsilon$, terminate.

Step 4: Solve the one-dimensional minimization problem (17), (18) and compute the new current solution:

$$\bar{f}_{ikj} = \bar{f}_{ikj}\ (1-\lambda^*) + \lambda^*\ \bar{y}_{ikj} \tag{20}$$

where $\lambda^*$ is the optimal step length.

Return to step 2.

## IV Numerical application: The Lindenwold High Speed Line Study

The particular case of the model presented in the previous section when there is only one final destination was implemented on the CDC Cyber 173 of the Université de Montréal and tested on data of the Lindenwold High Speed Line Study. This line links several southern New Jersey suburban communities to the Central Business District of Philadelphia and uses the park'n ride concept to attract commuters to transit. Two license plate surveys of commuters using the line were made in April and November 1970 in the context of a study done at the University of Pennsylvania[1]. The

1. See [2], [6] and [7] for more details on the surveys and the findings of that study.

total parking capacity of all stations was 5900 spaces for the April survey and increased by 2350 for the November survey. The origins i correspond to 687 census blocks. There are 7 stations k and one final destination (the CBD). The car-occupancy factor is taken as uniform and equal to 1. No data was available about car-occupancy since the survey was a "license-plate" survey. In 6 stations out of 7 there is free parking as well as pay-parking. In the 7th station (Broadway) there is only pay-parking. In the numerical tests of model (M1), pay-parking and free-parking were pooled and the parking fee disregarded.

The cost function used is:

$$c_{ik} = 2(v_a t + a)\, D_{ik} + F_k \qquad (21)$$

$c_{ik}$ = travel cost from census block i to station k.

$v_a$ = value of time = \$.042/min

$t$ = inverse of speed = 2.4 min/mile (25 miles/hr)

$a$ = auto operating cost/mile = \$.055/mile

$D_{ik}$ = distance in miles from census block i to station k

$F_k$ = round trip fare from station k to Philadelphia CBD

Another cost function $c'_{ik} = c_{ik} + \frac{v_a}{3} B_k$ (with $B_k$ = round trip travel time on the High Speed Line) was also used but the results obtained with it were not better than the results obtained with $c_{ik}$, and are not reported here.

The Golden Section Method was used to solve the one-dimensional minimization problem in the Frank and Wolfe algorithm and the transportation problem of linear programming was solved by a code due to B. Harris [18].

$\theta$ was calibrated on the first survey and the calibrated value used for prediction on the basis of the data of the second survey. Reciprocally the prediction for the first survey was done with a value of $\theta$ calibrated on the data of the second survey. The calibrated values of $\theta$ are:

| Survey | $\hat{\theta}$ |
|---|---|
| April | 6.0 |
| November | 5.7 |

It can be checked that the variations of $\theta$ are small and it may be considered constant between the 2 surveys. This fact justifies the assumption of a constant $\theta$ for prediction purposes in the short term.

For calibrating $\theta$ the total execution time varied between 158.0 sec. and 181.7 sec. Such execution times included 5 calls to the Frank and Wolfe method during the dichotomic search method and each call to Frank and Wolfe included 3 iterations (i.e. 3 calls to the transportation problem). The execution time for a call to the Frank and Wolfe algorithm during calibration varied between 30.1 sec. and 38.4 sec. For the solution of (M1), i.e. for predicting station choice on the basis of a given $\hat{\theta}$, the execution time varied between 57.4 sec. and 65.6 sec. In each case an $\varepsilon$-optimal solution was obtained in 6 iterations with $\varepsilon = 10^{-3}$. Table 1 compares the observed parking utilization (i.e. the station choices) to the predicted parking utilization for each survey. Examination of table 1 shows that the predictions on the basis of the new parking capacities of the second survey and on the basis of $\hat{\theta} = 6.0$ as calibrated on the first survey, produce a percentage error that varies between 0.0% and 8.4% for 6 of the

7 stations. On the other hand, the prediction for Broadway presents a percentage error of 68.9%. This high discrepancy is not as significant as it appears when absolute values are considered. The high relative error for Broadway can be explained by the fact that Broadway attracts a smaller number of commuters than the other stations. The error due to the small absolute value of trips is compounded by the fact that the numerical tests did not distinguish between pay-parking and free-parking. Since Broadway is the only station without free-parking, the cost input to the model was systematically lower than the real cost. A more refined prediction would take into account the cost of parking and distinguish between free-parking and pay-parking. This means that one should create two dummy stations for each real station, one corresponding to each parking type. There would be then 13 "stations" instead of 7. This has the consequence of doubling the size of the transportation problem in the Frank and Wolfe algorithm. This test was not yet made.

## V Extensions of the model to modal choice: private automobile versus transit, kiss'n ride versus park' ride

### A. Introduction

The model that was presented and tested in the preceding section predicts the allocation of park'n ride trips among competing stations of a rail transit line as a function of transport costs and parking capacities. However, from a transportation policy point of view, the interesting question is to determine how many commuter trips can be diverted from the private automobile to transit. This means that the choice of mode "car vs transit" must be included in a more relevant predictive model. Also one should be

| | | Lindenwold | Ashland | Haddonfield | Westmount | Collingswood | Perry Avenue | Broadway |
|---|---|---|---|---|---|---|---|---|
| FIRST SURVEY | Observed parking utilization | 1224 | 765 | 964 | 733 | 644 | 866 | 55 |
| | Predicted parking utilization | 1292 | 713 | 913 | 741 | 580 | 887 | 125 |
| | Percentage deviation | 5.5 | 6.8 | 5.3 | 1.1 | 9.9 | 2.4 | 127.3 |
| SECOND SURVEY | Observed parking utilization | 1375 | 866 | 1079 | 878 | 736 | 1049 | 74 |
| | Predicted parking utilization | 1481 | 793 | 1049 | 852 | 736 | 1021 | 125 |
| | Percentage deviation | 7.7 | 8.4 | 2.8 | 3.0 | 0.0 | 2.7 | 68.9 |

Table 1: Observed vs predicted parking occupations

able to distinguish between park'n ride and kiss'n ride. Hence the following models, which are direct extensions of (M1). In the first model, it is assumed that there is no congestion on the automobile network. In the second model, congestion is introduced.

B. Model predicting modal choice and station choice when congestion is disregarded on the automobile network

As in the model of section II, $i \in I$ and $j \in J$ are origins and final destinations of trips, and $k \in K$ are the stations on the rail transit line. The superscript au will represent travel by car.

$m = 1$ represents park'n ride.

$m = 2$ represents kiss'n ride.

$g_{ij}^{au}$ is the number of trips by automobile from i to j.

$f_{ikj}^{m}$ is the number of trips by transit using station k and park'n ride ($m = 1$) or kiss'n ride ($m = 2$) from i to j.

$c_{ikj}^{m}$ is the cost of travel by park'n ride or kiss'n ride between i and j, through station k.

For park'n ride, $c_{ikj}^{m}$ includes the cost of parking, while for kiss'n ride it includes the value of time spent travelling by the commuter's spouse.

$d_{ij}^{au}$ is the cost of travel by car from i to j.

The variables $f_{ikj}^{m}$ and $g_{ij}^{au}$ are required to be the solution to the following minimization problem:

$$(M2)\quad \text{Min} \sum_i \sum_j g_{ij}^{au} [\ell n\, g_{ij}^{au} + \theta\, d_{ij}^{au}] \qquad (22)$$

$$+ \sum_m \sum_i \sum_k \sum_j f_{ikj}^{m} [\ell n\, f_{ikj}^{m} + \theta\, c_{ikj}^{m}]$$

$$\text{s.t.} \quad \sum_j g^{au}_{ij} + \sum_m \sum_j \sum_k f^m_{ikj} = O_i \qquad \text{all } i \qquad (23)$$

$$\sum_i g^{au}_{ij} + \sum_m \sum_i \sum_k f^m_{ikj} = D_j \qquad \text{all } j \qquad (24)$$

$$\sum_i \sum_j f^1_{ikj} \leq \gamma_k T_k \qquad \text{all } k \qquad (25)$$

$$f^m_{ikj} \geq 0 \quad g^{au}_{ij} \geq 0 \qquad (26)$$

The objective function is convex in $f^m_{ikj}$ and $g^{au}_{ij}$ and we set $f^m_{ikj}$ $\ell n$ $f^m_{ikj} = 0$ at $f^m_{ikj} = 0$ and $g^{au}_{ij}$ $\ell n$ $g^{au}_{ij} = 0$ at $g^{au}_{ij} = 0$.

The Kuhn-Tucker conditions are necessary and sufficient conditions for an optimal solution. After a few suitable modifications, these become:

$$f^{*1}_{ikj} = \exp(-v_k) \exp(-1-\alpha_i-\beta_j) \exp(-\theta\, c^1_{ikj}) \qquad (27)$$

$$f^{*2}_{ikj} = \exp(-1-\alpha_i-\beta_j) \exp(-\theta\, c^2_{ikj}) \qquad (28)$$

$$g^{*au}_{ij} = \exp(-1-\alpha_i-\beta_j) \exp(-\theta\, d^{au}_{ij}) \qquad (29)$$

$$v_k > 0 \Rightarrow \gamma_k T_k = \sum_i \sum_j f^1_{ikj} \qquad (30)$$

$$\gamma_k T_k - \sum_i \sum_j f^1_{ikj} > 0 \Rightarrow v_k = 0 \qquad (31)$$

Station choice

For kiss'n ride and a given origin-destination pair, station choice is given by

$$\frac{f^{*2}_{ikj}}{\sum_k f^{*2}_{ikj}} = \frac{\exp(-\theta\, c^2_{ikj})}{\sum_k \exp(-\theta\, c^2_{ikj})} \qquad (32)$$

This is a logit model.

For park'n ride and a given origin-destination pair, station choice is given by

$$\frac{f^{*1}_{ikj}}{\sum_k f^{*1}_{ikj}} = \frac{\exp(-v_k)\exp(-\theta\, c^1_{ikj})}{\sum_k \exp(-v_k)\exp(-\theta\, c^1_{ikj})} \tag{33}$$

This is a "weighted" logit model analogous to (8)

Choice of the mode of access: park'n ride versus kiss'n ride

For a given origin-destination pair, and a given station, the choice between park'n ride and kiss'n ride is given by:

$$\frac{f^{*1}_{ikj}}{f^{*1}_{ikj} + f^{*2}_{ikj}} = \frac{1}{1 + \exp[v_k + \theta\,(c^1_{ikj} - c^2_{ikj})]} \tag{34}$$

The only differences between $c^1_{ikj}$ and $c^2_{ikj}$ are the parking fee $\Pi_k$ and the value of the time spent by the commuter's spouse in traveling between origin i and station k, $t_{ik}$. Since $t_{ik}$ is obtained by multiplying distance traveled by the value of time, one can utilize different values of time for spouses in different zones depending on the socio-economic and demographic characteristics of the zones. Equation (34) can be restated as:

$$\frac{f^{*1}_{ikj}}{f^{*1}_{ikj} + f^{*2}_{ikj}} = \frac{1}{1 + \exp[v_k + \theta\,(\Pi_k - t_{ik})]} \tag{35}$$

Equation (35) clearly shows that the choice between park'n ride and kiss'n ride is the result of a tradeoff between parking capacity, parking fee and value of time of the commuter's spouse.

Modal choice: car versus transit

For a given origin-destination pair the choice between car and transit is given by

$$\frac{g^{*au}_{ij}}{g^{*au}_{ij}+\sum_k \sum_m f^{*m}_{ikj}} = \frac{\exp\ (-\theta d^{au}_{ij})}{\exp\ (-\theta d^{au}_{ij})+\sum_k \exp(-v_k)\ \exp(-\theta c^{1}_{ikj}) + \sum_k \exp(-\theta c^{2}_{ikj})} \qquad (36)$$

Solution algorithm

For a given value of $\theta$, the problem (M2) consists in minimizing a convex function subject to linear constraints. It can be solved again by an adaptation of the Frank and Wolfe algorithm.

C. Model predicting modal choice and station choice when there is congestion on the automobile network

In reality there is congestion on the automobile network and it is important to determine the impact of congestion on the choice between car and rail transit. This is done in the following model which is analogous to the model developed by Florian and Nguyen [13] for the combined modal choice, trip distribution and traffic assignment. However Florian and Nguyen's model was adapted to the choice between the bus mode and the car mode, while the present model is adapted to the choice between a rail transit line and the car mode and includes station choice as well as the choice between park'n ride and kiss'n ride, for access to the station. The model presented here can also be considered as an extension of the model (M2). It makes the assumption that there is no congestion due to car trips between trip origins i and rail transit stations k. Only automobiles that are destined to final

destinations contribute to congestion in the model. This assumption is valid only if the stations on the transit line are suburban, as in the Lindenwold High Speed Line case, but not if they are urban. In the latter case cars destined to stations would have a non-negligible effect on traffic congestion. In addition Wardrop's user equilibrium assumption is made for assigning trips to arcs of the road network[1]. The assumption is made of a uniform car-occupancy factor $\gamma$. The congestion functions are convex functions of the flows $v_a$ on the arcs a of the automobile network: $s_a(v_a)$, $h^{au}_{p,ij}$ is the path flows $v_a$ on path p between i and j on the automobile network. Except for additional notation, the notation is the same as for model (M2).

Then $g^{au}_{ij}$, $f^{m}_{ikj}$ and $h^{au}_{p,ij}$ are required to be the solution to the following minimization problem.

$$\text{(M3)}\quad \text{Min } \frac{1}{\theta} \sum_i \sum_j g^{au}_{ij} \ln g^{au}_{ij} + \sum_a \int_0^{v_a} s_a(x)\,dx$$

$$+ \sum_m \sum_i \sum_k \sum_j f^{m}_{ikj} \left[\frac{1}{\theta} \ln f^{m}_{ikj} + c^{m}_{ikj}\right] \qquad (37)$$

$$\text{s.t.} \quad \sum_j g^{au}_{ij} + \sum_m \sum_j \sum_k f^{m}_{ikj} = O_i \qquad \text{all i} \qquad (38)$$

$$\sum_i g^{au}_{ij} + \sum_m \sum_i \sum_k f^{m}_{ikj} = D_j \qquad \text{all j} \qquad (39)$$

$$\sum_i \sum_j f^{l}_{ikj} \le \gamma\, T_k \qquad \text{all k} \qquad (40)$$

$$\sum_p h^{au}_{p,ij} = g^{au}_{ij} \qquad \text{all i,j} \qquad (41)$$

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1. See[21], [22] and [23] for a discussion of these concepts.

$$v_a = \sum_i \sum_j \sum_p \delta_{ap,\,ij} \cdot h^{au}_{p,ij}$$

where $\delta_{ap,ij} = \begin{cases} 1 \text{ if a belongs to path p between i and j} \\ 0 \text{ otherwise} \end{cases}$

$$g^{au}_{ij} \geq 0,\ f^{m}_{ikj} \geq 0,\ h_{p,ij} \geq 0 \qquad (42)$$

Constraint (41) is a conservation of flows constraint.

We set $g^{au}_{ij}\ \ell n\ g^{au}_{ij} = 0$ at $g^{au}_{ij} = 0$

and $f^{m}_{ikj}\ \ell n\ f^{m}_{ikj} = 0$ at $f^{m}_{ikj} = 0$

The objective function is convex in the variables $g^{au}_{ij}$, $f^{m}_{ikj}$ and $h^{au}_{p,ij}$.

The Kuhn-Tucker conditions imply:

$$g^{*au}_{ij} = \exp(-1-\alpha_i-\beta_j)\exp(-\theta\, U^{au}_{ij}) \qquad (43)$$

$$f^{*1}_{ikj} = \exp(-v_k)\exp(-1-\alpha_i-\beta_j)\exp(-\theta\, c^{1}_{ikj}) \qquad (44)$$

$$f^{*2}_{ikj} = \exp(-1-\alpha_i-\beta_j)\exp(-\theta\, c^{2}_{ikj}) \qquad (45)$$

$$h^{au}_{p,ij} > 0 \Rightarrow \sum_a s_a\,(v^*_a)\,\delta_{ap,ij} - U^{au}_{ij} = 0 \qquad (46)$$

$$h^{au}_{p,ij} = 0 \Rightarrow \sum_a s_a\,(v^*_a)\,\delta_{ap,ij} - U^{au}_{ij} \geq 0 \qquad (47)$$

$$v_k > 0 \Rightarrow \gamma\, T_k = \sum_i \sum_j f^{1}_{ikj} \qquad (48)$$

$$\gamma\, T_k - \sum_i \sum_j f^{1}_{ikj} > 0 \Rightarrow \quad v_k = 0 \qquad (49)$$

In the preceding constraints $U^{au}_{ij}$ is the dual variable associated with the conservation of flows constraint; (46) and (47) are Wardrop's user equilibrium conditions. By comparing (43), (44) and (45) to the conditions (27 to (29) one notices that they

are equivalent if one replaces $d_{ij}^{au}$, a constant in model (M2) by $U_{ij}^{au}$, the equilibrium dual variable representing the cost of travel by automobile between i and j, in model (M3). With the latter qualification, (M2) and (M3) produce the same expressions for station choice and modal choice (See Equations (32), (33), (34), (35) and (36)).

Solution algorithm: For a given value of $\theta$, (M3) consists in minimizing a convex function subject to linear constraints. It can be solved by an adaptation of the Frank and Wolfe technique.

## VI Conclusion

This paper presented an entropy maximization model for predicting station choice in a park'n ride context, with explicit capacity constraints for parking. A particular case of the model was implemented and tested. Comparison between observed and predicted choices for these preliminary tests shows that the method gives good predictions and is computationally feasible for problems of a realistic size (here 687 origins, 7 stations and one final destination). One should note however that in the case of one final destination special advantage was taken of the special structure of the linear programming subproblem of the Frank and Wolfe algorithm. General linear programming codes would be necessary if there were several final destinations and they are less efficient than codes for the transportation problem.

Two models predicting modal choice, choice of the mode of access as well as station choice were presented. However no data was available to test them. From a transportation policy point of view however they would be

more useful than the station choice model since they could help in determining the policy on parking capacity, parking fee, station location or number of stations, which is adequate to lead more commuters to use rail transit rather than the private automobile, thus relieving urban traffic congestion. This is all the more important that many large North American metropolises have inherited, like Philadelphia, railroad tracks, that are not used as much as they could for commuting.

## ACKNOWLEDGEMENTS

This paper owes much to the encouragement and collaboration of Michael Florian. I want to thank also David E. Boyce for kindly supplying me with the data of the Lindenwold High Speed Line Study, Britton Harris for supplying me with an extremely efficient code for the transportation problem of linear programming and Guy Saint-Pierre for his expert collaboration in the computer implementation of the model and the numerical tests.

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## APPENDIX

### The Frank and Wolfe algorithm [15]

Consider the problem of minimizing a convex function subject to linear constraints:

$$\text{Min } Z(x) \text{ , subject to } A X \leq b \text{ , } X \geq 0.$$

The detailed steps of the Frank and Wolfe algorithm for solving this problem are:

**Step 1** Given a feasible solution $X^1$ , set $\ell = 1$.

**Step 2** Determine $Y^\ell$ that minimizes

$$\nabla Z(X^\ell)\, Y \text{ , subject to } A Y \leq b \text{ , } Y \geq 0 .$$

**Step 3** Set the descent direction $d^\ell = Y^\ell - X^\ell$ .

If $| \nabla Z(X^\ell)\, d^\ell | \leq \varepsilon$ , terminate, where $\varepsilon$ is a suitable convergence parameter. ($X^\ell$ is the optimal solution).

**Step 4** Find the optimal step length $\lambda^\ell$ that minimizes

$$Z(X^\ell + \lambda\, d^\ell) \quad \text{for} \quad 0 \leq \lambda \leq 1 .$$

**Step 5** Revise the current solution

$$X^{\ell+1} = X^\ell + \lambda^\ell\, d^\ell ;$$

set $\ell = \ell + 1$ and return to Step 2.